The Logic of Comparative Theory Evaluation

1. Critique of Schaffner's account
2. Bayesian analysis of ad hocness
3. The Case of Multiple Predictions
4. Quantitative predictions
5. Bayesian Model for MSRP
6. An alternative approach
   – The Methodological decision

Bayes's Theorem

$$P(T/\iota \,\&\, e) = \frac{P(T/\iota) \cdot P(e/T \,\&\, \iota)}{P(e/\iota)}$$

$$P(e/\iota) = \sum_{i=1}^{n} P(T_i/\iota) \cdot P(e/T_i \,\&\, \iota)$$

or

$$P(T/\iota \,\&\, e) = \frac{P(T/\iota) \cdot P(e/T \,\&\, \iota)}{P(e/T \,\&\, \iota)\, P(T/\iota) + P(e/\sim T \,\&\, \iota) \cdot P(\sim T/\iota)}$$

Enhancement Ratio

$$\Omega = P(T/\nu \& e)/P(T/\nu)$$

Define $x = P(T/\nu)$

$\qquad\quad\; \varepsilon = P(e/\sim T \& \nu)$

then $$\Omega = \frac{1}{x + \varepsilon(1-x)}$$

Multiple Predictions

$$P_n = \frac{1}{1-\varepsilon^n + \varepsilon^n/x}$$

$$P(e_{n+1}) = \varepsilon + \frac{1-\varepsilon}{1-\varepsilon^n + \varepsilon^n/x}$$

Simple model for ε

Theory describes M events, each with N possible results.

n = Total number of theories

m = number of theories which make a particular true prediction e

then
$$\begin{cases} n = \overline{\overline{\{f : \{M\} \to \{N\}\}}} = N^M \\ m = N^{M-1}. \end{cases}$$

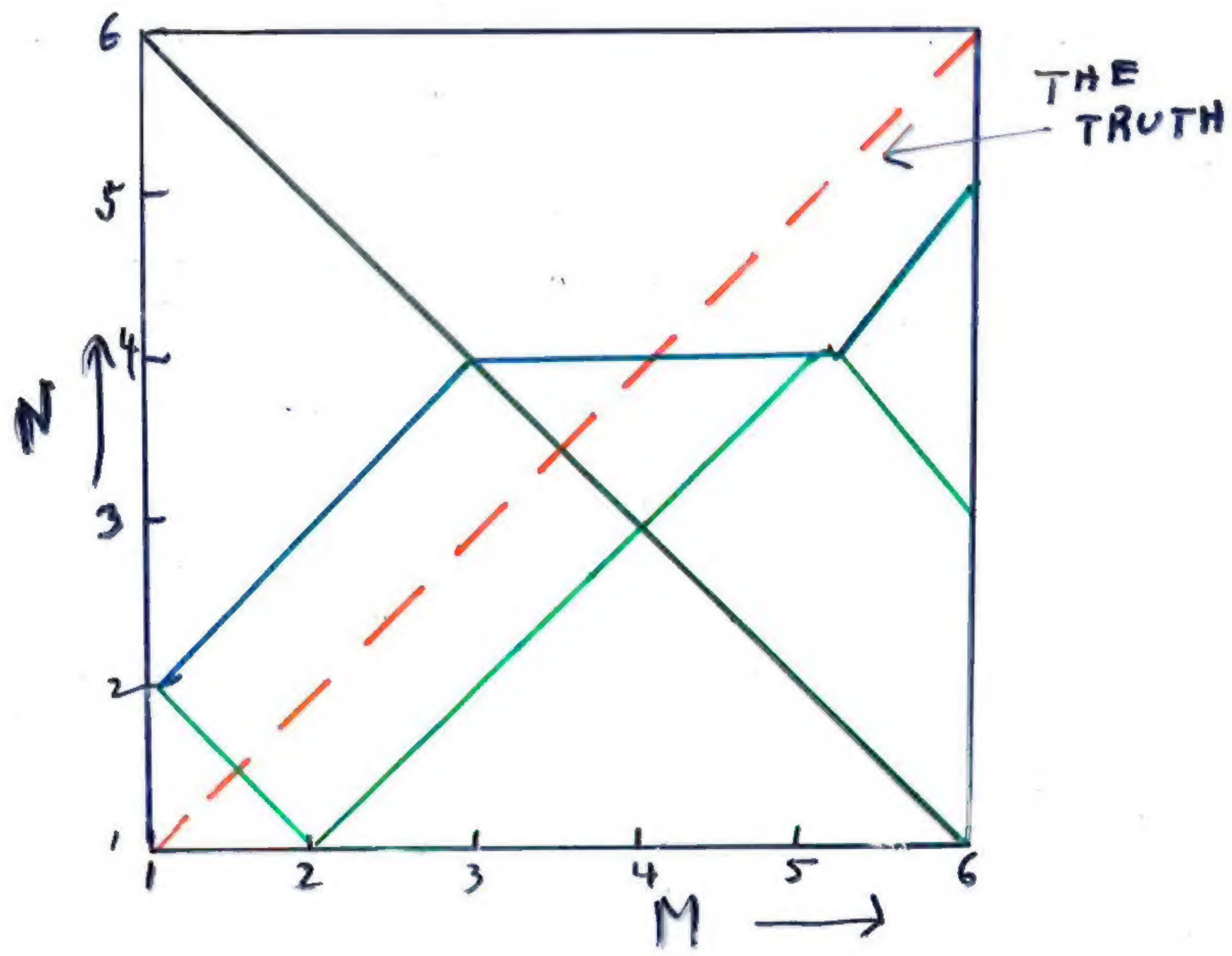
Take $P(T_1|\mathcal{Z}) = x$, $P(T_i|\mathcal{Z}) = \frac{1-x}{n-1}$, $i \neq 1$

$$P(e|\mathcal{Z}) = P(e/T_1 \& \mathcal{Z})\, P(T_1|\mathcal{Z}) + \sum_{i \neq 1} P(e/T_i \& \mathcal{Z}) . P(T_i|\mathcal{Z})$$

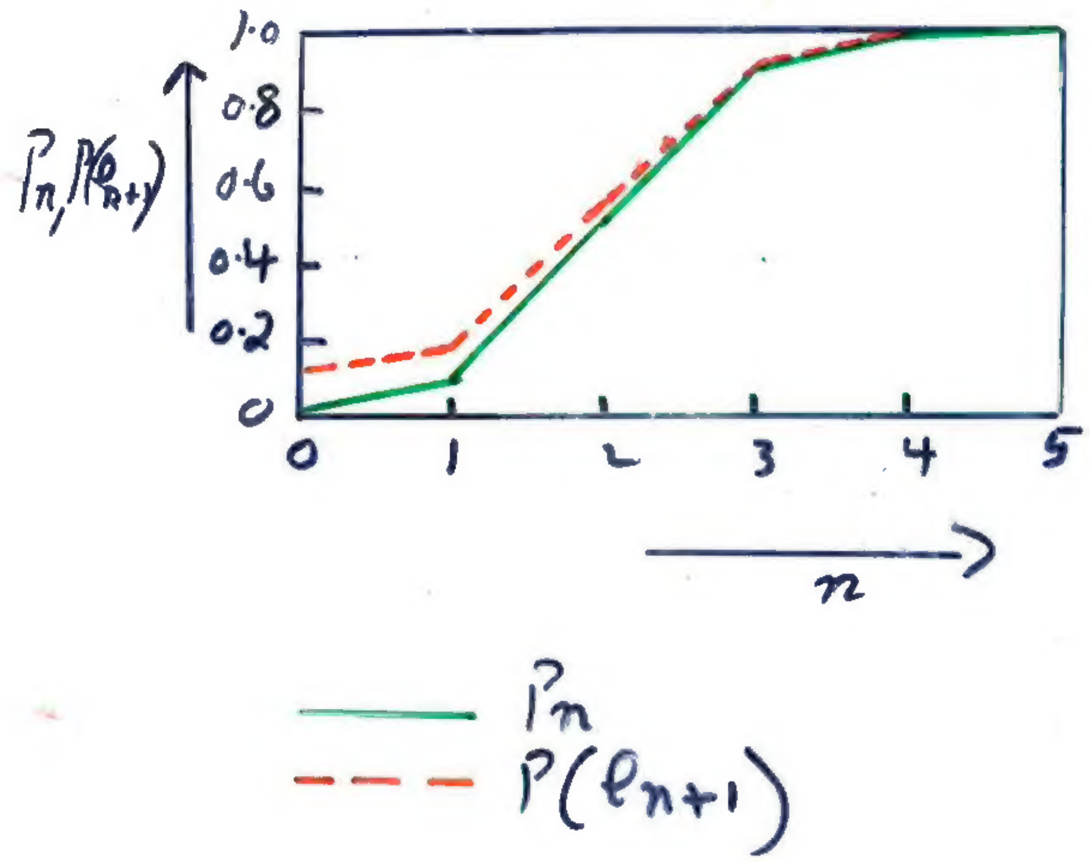
$$= x + (m-1) \cdot \frac{1-x}{n-1}$$

Compare with $P(e|\mathcal{Z}) = x + \varepsilon\,(1-x)$

Hence $\varepsilon = \frac{m-1}{n-1} = \frac{N^{M-1}-1}{N^M-1} \simeq \frac{1}{N}$


THE TRUTH
N
M
1
2
3
4
5
6

Multiple predictions



$$\left.\begin{aligned} x &= 0.01 \\ \varepsilon &= 0.1 \end{aligned}\right\}$$

Dorling analysis for confirmation

Expand in Bayes' theorem

$$P(e/c \,\&\, b) = P(e/c \,\&\, B \,\&\, b) \cdot P(B/c \,\&\, b) + P(e/c \,\&\, \sim B \,\&\, b) \cdot P(\sim B/c \,\&\, b)$$

Write $P(c/b) = x$, $P(B/c \,\&\, b) = y$

Assume $P(e/c \,\&\, \sim B \,\&\, b) = P(e/\sim c \,\&\, b)$

$= P(e/\sim c \,\&\, B \,\&\, b) = P(e/\sim B \,\&\, b)$

$= k$ say.

then

$$\left.\begin{aligned} r_c &= \frac{y + k(1-y)}{xy + k(1-xy)} \\ r_B &= \frac{x + k(1-x)}{xy + k(1-xy)} \end{aligned}\right\}$$

Dorling analysis for refutation

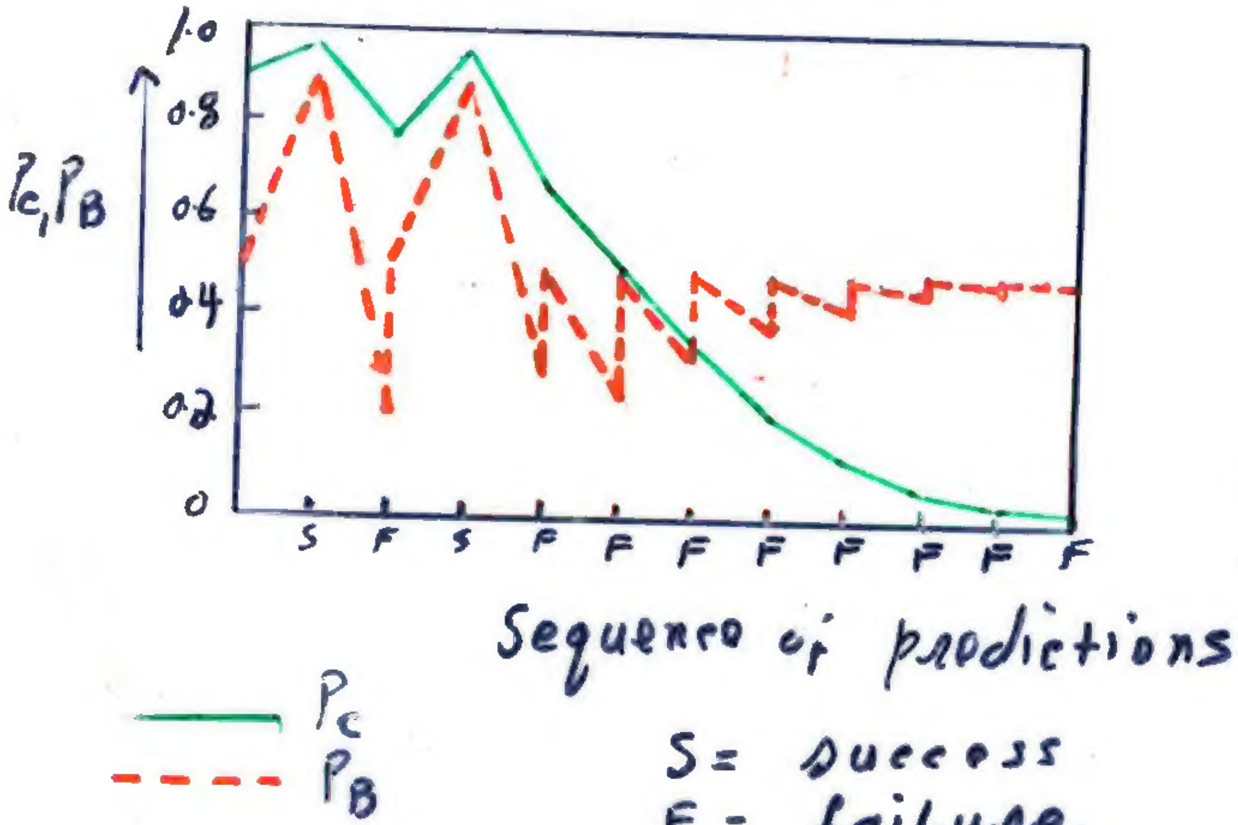
$$r_C = \frac{1-y}{1-xy}$$

$$r_B = \frac{1-x}{1-xy}$$

For asymmetric effect of refutation take x close to unity, $y \ll$ unity i.e. $1-y \gg 1-x$.

The Darling Model

$x = 0.9,\ y = 0.5$
$k = 0.1$



S = success
F = failure

Methodological Decisions

Refutation $T = C \,\&\, B$

$$P(T/b\,\&\,e) = P(C/b\,\&\,e) \times P(B/b\,\&\,e) = 0$$

choose $P(B/b\,\&\,e) = 0$

Confirmation

$$P(T/b\,\&\,e) = \Omega \times P(T/b)$$

write $\Omega = \Omega_C \times \Omega_B$

where
$$P(C/b\,\&\,e) = \Omega_C \times P(C/b)$$
$$P(B/b\,\&\,e) = \Omega_B \times P(B/b)$$

How do we **choose** Ω_C & Ω_B ?

The Factorization Constraints

$$\Omega \cdot A \cdot B = (\Omega_A \cdot A) \times (\Omega_B \cdot B)$$

(1) $\Omega_A(1) = \Omega_B(1) = 1.$

(2) $\Omega_A\left(\frac{1}{AB}\right) = \frac{1}{A}$, $\Omega_B\left(\frac{1}{AB}\right) = \frac{1}{B}$

(3) $\Omega_A(\Omega)$ & $\Omega_B(\Omega)$ monotonic increasing functions of Ω in range $(1, 1/AB)$

(4) $\Omega_A \cdot A \geqslant \Omega_B \cdot B$ for all Ω in range $(1, 1/AB)$

(5) $\Omega_A \cdot \Omega_B = \Omega.$

(6) Write $\Omega_A(\Omega) = f(A, B, \Omega)$

$$f(A, B, \Omega'\Omega'') = f(A, B, \Omega') \times f\left[f(A, B, \Omega') \cdot A, \frac{\Omega}{f(A, B, \Omega')} \cdot B, \Omega''\right]$$

Solution satisfying conditions (1)..(6) is

$$\Lambda_A = \frac{\Lambda(1-AB)}{1-A+A(1-B)\Lambda}.$$

This does not satisfy condition

(7) $\Lambda_A = \Lambda_B = \sqrt{\Lambda}$ for $A = B$

Darling soln for conditions (1)...(7) is

$$\Lambda_A = \frac{B + \beta(1-B)}{AB + \beta(1-AB)}$$

β is unique root in range $0 < \beta < 1$ of quadratic equation

$$\beta^2\{(1-A)(1-B) - (1-AB)^2\Lambda\} + \beta\{A+B-2AB-2AB(1-AB)\Lambda\} + AB(1-AB\Lambda) = 0$$

The Memory Model

$$P_c(n_1, n_2 \cdots n_m) = P_c'(n_1 \cdots n_{m-1}) \times \Theta\left(\sqrt{x}, P_c'(n_1 \cdots n_{m-1}), \Omega^{(n_m)}\right)$$

where $P_c'(n_1 \cdots n_{m-1}) = F(m-1) \times P_c(n_1 \cdots n_{m-1})$

and $\Omega^{(n_m)} = \Phi\left(\varepsilon, n_m, \sqrt{x} \cdot P_c'(n_1 \cdots n_{m-1})\right)$

$$\Omega^{(n_1)} = \Phi(\varepsilon, n_1, x)$$

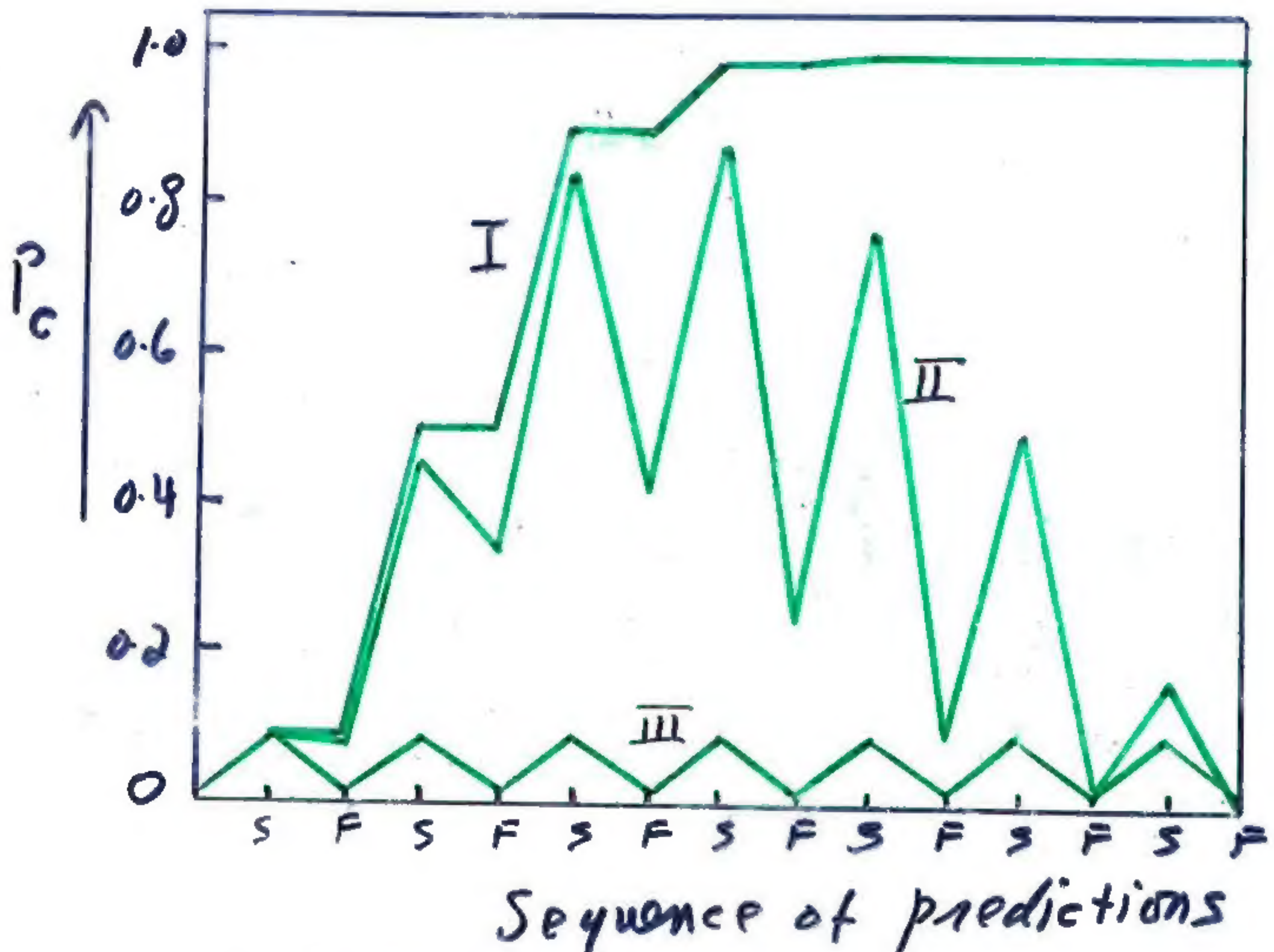
with $P_c(n_1) = \sqrt{x}\; \Theta\left(\sqrt{x}, \sqrt{x}, \Omega^{(n_1)}\right)$

and $\Theta(A, B, C) = \dfrac{C(1 - AB)}{1 - B + B(1-A)C}$

$$\Phi(A, B, C) = \left(C(1 - A^B) + A^B\right)^{-1}$$

$$F(m) = \left(1 + \exp(m - m_0)\right)^{-1}$$

The Life & Death of a Research Programme



I Pure Lakatos
II Pure Lakatos plus memory
III. Pure Popper

$\left.\begin{array}{l} \sqrt{x} = 0.01 \\ \varepsilon = 0.1 \\ m_0 = 3 \end{array}\right\}$